A Quantum Theory of Speciation

Kerry M. Soileau

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ABSTRACT

We propose a theoretical model of quantum speciation among elements of a finite dimensional Hilbert space. The potential for species diversity and the current environment are represented by linear operators satisfying a compatibility criterion. A method for calculating probabilities of production of individuals is defined.

INTRODUCTION

Let H be a Hilbert space H with inner product $\langle \cdot,\cdot \rangle$ and finite dimension n. We say that the ordered pair (A,B) is <u>compatible</u> if A and B are linear operators on H, B is Hermitian and the composition AB has all real eigenvalues and a unique largest eigenvalue. By C_H we mean the collection of compatible ordered pairs.

Fix $(E,S) \in C_H$. Let the environment be represented by E, and the species by S. The interaction of the species with the environment is represented by the linear operator $R = ES$. Let the unit eigenvectors of S be denoted by $V(S) = \{s_1, s_2, s_3, \cdots, s_n\}$. This set represents the "individuals" genetically possible for the species represented by S. Note that $V(S)$ forms an orthonormal basis for H. Let the unit eigenvectors of R be denoted by $V(R)$, and let

$r \in V(R)$ be the eigenvector with largest eigenvalue. The probability of production of the individual s_i is defined to be $|\langle r, s_i \rangle|^2$ for each $i = 1, 2, 3, \cdots, n$.

EXAMPLE

Take the Hilbert space to be $\mathbb{R}^8$ with the usual topology and inner product. Let the environment be represented by the matrix

$$E = \begin{pmatrix} .587466 & .383252 & -495393 & .457811 & -.877964 & .533748 & .582221 & .618521 \\ .383252 & .251454 & -.323737 & .299177 & -.573745 & .348802 & .380478 & .404201 \\ -.495393 & -.323737 & 3.01535 & -.386717 & -1.39056 & -.451316 & -.49136 & -.525113 \\ .457811 & .299177 & -.386717 & .358379 & -.685362 & .416658 & .454497 & .482834 \\ -.877964 & -.573745 & -1.39056 & -.685362 & 3.07425 & -.799734 & -.87093 & -.929968 \\ .533748 & .348802 & -.451316 & .416658 & -.799734 & .487769 & .529884 & .562921 \\ .582221 & .380478 & -.49136 & .454497 & -.87093 & .529884 & .578106 & .614045 \\ .618521 & .404201 & -.525113 & .482834 & -.929968 & .562921 & .614045 & .658324 \end{pmatrix},$$

and let the species be represented by the matrix

$$S = \begin{pmatrix} 4.62403 & -.322869 & -.943941 & -.408346 & -.367027 & -.203681 & -1.32341 & .0671462 \\ -.322869 & 7.96912 & -.090276 & -.0390531 & -.0351014 & -.0194795 & -.126568 & .00642168 \\ -.943941 & -.090276 & 6.10552 & .434195 & .785226 & -.2763 & -.7169 & .0493878 \\ -.408346 & -.0390531 & .434195 & 5.32538 & .123354 & -.10429 & -.901593 & -.153763 \\ -.367027 & -.0351014 & .785226 & .123354 & 2.96294 & .247461 & .611623 & .0879236 \\ -.203681 & -.0194795 & -.2763 & -.10429 & .247461 & 3.62061 & -.00487578 & .939697 \\ -1.32341 & -.126568 & -.7169 & -.901593 & .611623 & -.00487578 & 4.03868 & -.116322 \\ .0671462 & .00642168 & .0493878 & -.153763 & .0879236 & .939697 & -.116322 & 1.35372 \end{pmatrix}$$

The interaction of the species with the environment is given by the matrix

$$R = ES = \begin{pmatrix} 2.35793 & 2.84206 & -4.6387 & 1.18407 & -2.62038 & 2.25563 & .856288 & 1.141 \\ 1.53755 & 1.86545 & -3.03084 & .774009 & -1.7122 & 1.47416 & .560318 & .745597 \\ -3.65727 & -2.56064 & 18.0983 & -.135942 & -2.06533 & -3.1548 & -3.88812 & -1.02702 \\ 1.83664 & 2.2188 & -3.61992 & .929958 & -2.04513 & 1.76086 & .668573 & .890485 \\ -2.15729 & -4.12455 & -4.69376 & -2.48182 & 7.46242 & -2.35872 & 1.3243 & -1.66475 \\ 2.14204 & 2.5869 & -4.22474 & 1.07751 & -2.38641 & 2.06025 & .780414 & 1.04016 \\ 2.33547 & 2.82173 & -4.601 & 1.17609 & -2.59863 & 2.23955 & .851902 & 1.13274 \\ 2.4863 & 2.99816 & -4.91025 & 1.24665 & -2.77667 & 2.3845 & .903232 & 1.21092 \end{pmatrix}$$

The eigenvalues of the matrix R are given by

$$\lambda(R) = \{22.6344,\ 12.1662,\ .0154741,\ .00676189,\ .00604594,\ .00505311,\ .00275748,\ .000398656\}$$

The eigenvectors of the matrix R are given by

$$V(R) = \left\{ \begin{array}{l} (-.262413,\ -.171502,\ .806404,\ -.204812,\ -.0871612,\ -.238993,\ -.260346,\ -.277481), \\ (.24918,\ .162898,\ .284726,\ .194527,\ -.781302,\ .226931,\ .247237,\ .263453), \\ (-.296877,\ -.358511,\ -.238824,\ -.313701,\ -.397312,\ -.152791,\ -.392298,\ .5411), \\ (.274745,\ -.523329,\ .119413,\ .470479,\ .0654829,\ .499673,\ .264841,\ -.301547), \\ (-.464368,\ .0682246,\ .075601,\ -.296825,\ .0802433,\ .817321,\ .0696652,\ -.0811256), \\ (-.699415,\ .0496956,\ -.208304,\ .539336,\ -.105897,\ -.232158,\ -.189755,\ .270105), \\ (.0279293,\ .0443105,\ -.0826777,\ -.0280428,\ -.0627183,\ .325099,\ -.392075,\ -.852226), \\ (.111177,\ .0799963,\ .0225389,\ .08178,\ .38814,\ .0415225,\ -.495561,\ .759004) \end{array} \right\}$$

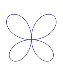
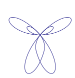
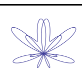
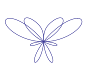
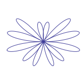
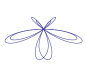
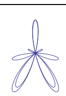
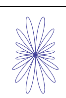
The eigenvector of R with largest eigenvalue is

$$(-.262413,\ -.171502,\ .806404,\ -.204812,\ -.0871612,\ -.238993,\ -.260346,\ -.277481).$$

The eigenvectors of the matrix S are given by

$$V(S)=\left\{\begin{array}{l}(.0952029,\ -.995458,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0),\\ (-.345654,\ -.0330574,\ .82615,\ .382476,\ .182412,\ -.0475414,\ -.122421,\ -.00988729),\\ (.558176,\ .0533825,\ -.0405465,\ .473144,\ -.196446,\ -.0886957,\ -.642993,\ -.0135071),\\ (.39819,\ .0380819,\ .478511,\ -.745893,\ -.00662532,\ -.0772517,\ -.218207,\ .0322277),\\ (.0295167,\ .0028229,\ .0155715,\ -.00841796,\ .152084,\ .913988,\ -.160965,\ .338222),\\ (-.499804,\ -.0477999,\ -.00603601,\ -.173625,\ -.771892,\ .075417,\ -.340249,\ .0213282),\\ (-.386619,\ -.0369752,\ -.292338,\ -.205905,\ .556007,\ -.162326,\ -.617925,\ -.0630506),\\ (-.0345529,\ -.00330455,\ -.0334439,\ .0296146,\ -.000591584,\ -.341312,\ .021189,\ .938007)\end{array}\right\}$$

The probabilities of production are as shown in the following table:

Individual		Probability of Production
	(.0952029, -.995458, 0, 0, 0, 0, 0, 0)	.021240
	(-.345654, -.0330574, .82615, .382476, .182412, -.0475414, -.122421, -.00988729)	.510263
	(.558176, .0533825, -.0405465, .473144, -.196446, -.0886957, -.642993, -.0135071)	.005740
	(.39819, .0380819, .478511, -.745893, -.00662532, -.0772517, -.218207, .0322277)	.244557
	(.0295167, .0028229, .0155715, -.00841796, .152084, .913988, -.160965, .338222)	.077054
	(-.499804, -.0477999, -.00603601, -.173625, -.771892, .075417, -.340249, .0213282)	.091183
	(-.386619, -.0369752, -.292338, -.205905, .556007, -.162326, -.617925, -.0630506)	.006876
	(-.0345529, -.00330455, -.0334439, .0296146, -.000591584, -.341312, .021189, .938007)	.043087

MOTIVATION

In quantum mechanics, observables are represented by self-adjoint operators on a Hilbert space. Thus in proposing a model of quantum speciation, it is natural to regard a species as a whole as some self-adjoint operator S. In the quantum mechanical setting, each possible measurement of an observable corresponds to a unit eigenvector and eigenvalue of this operator, so by analogy we regard each unit eigenvector of the species linear operator S to represent a possible individual. We postulate that each species will have only finitely many possible individuals, thus we assume that S, and also the Hilbert space, have finite dimension. Thus S is in fact Hermitian. We may then regard the eigenvalues of each unit eigenvector (i.e. individual) of S as representing the reproductive strength of that individual. We model the influence of the environment by means of a linear operator E which is composed with S to produce the resultant operator $R = ES$. We require that (E,S) be compatible, in the sense defined above, so that R will have all real eigenvalues and a unique largest eigenvalue.

The definition of probability of production was motivated by the following observation. If $\vec{\varphi}$ is a random vector in $\mathbb{R}^n$, how may we determine the unit vector $\hat{v} \in \mathbb{R}^n$ which maximizes the expectation value $E(\vec{\varphi} \cdot \hat{v})^2$? It's not difficult to show that this is accomplished by taking $\hat{v}$ to be an eigenvector with maximal eigenvalue of the matrix $\begin{pmatrix} E\varphi_1\varphi_1 & E\varphi_1\varphi_2 & \cdots & E\varphi_1\varphi_n \\ E\varphi_2\varphi_1 & E\varphi_2\varphi_2 & \cdots & E\varphi_2\varphi_n \\ \vdots & \vdots & \ddots & \vdots \\ E\varphi_n\varphi_1 & E\varphi_n\varphi_2 & \dots & E\varphi_n\varphi_n \end{pmatrix}$. The maximal value of $E(\vec{\varphi} \cdot \hat{v})^2$ is then equal to this eigenvalue. Thus in the case that there exists a random vector $\vec{\varphi}$ such that $R = \begin{pmatrix} E\varphi_1\varphi_1 & E\varphi_1\varphi_2 & \cdots & E\varphi_1\varphi_n \\ E\varphi_2\varphi_1 & E\varphi_2\varphi_2 & \cdots & E\varphi_2\varphi_n \\ \vdots & \vdots & \ddots & \vdots \\ E\varphi_n\varphi_1 & E\varphi_n\varphi_2 & \dots & E\varphi_n\varphi_n \end{pmatrix}$, $E(\vec{\varphi} \cdot \hat{v})^2$ is maximized for $\hat{v} = \hat{v}_{\max}$, where $\hat{v}_{\max}$ is the largest eigenvalue of R. We may express $\hat{v}_{\max}$ as a unique linear combination of the eigenvectors (individuals) of S, like so: $\hat{v}_{\max} = \sum_{i=1}^{N} (\hat{v}_{\max} \cdot \hat{s}_i)\hat{s}_i$. Again following the pattern seen in the quantum mechanical setting, we define the probability of "observing", i.e. producing the individual represented by $\hat{s}_i$ as $\left|\langle \hat{v}_{\max}, \hat{s}_i \rangle\right|^2 = (\hat{v}_{\max} \cdot \hat{s}_i)^2$. Although the motivation involves Hermitian operators R, this is not assumed in the definition of compatible operators.

REFERENCE

Carson, Hampton L., Chromosomal Tracers of Founder Events, *Biotropica*, Vol. 2, No. 1 (Jun., 1970), pp. 3-6.