A Noniterative Solution to the Glint Problem

Kerry M. Soileau

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I. Introduction

Earth observation from space is sometimes enhanced by the reflection ("glint") of the Sun off of the ocean. Consequently, it is of operational interest to know the location of the glint point, the point on the Earth marking the center of the Sun's glint.

Figure 1 exhibits the trigonometry involved. A cross-section of the Earth is shown; the plane of the cross-section has been chosen so that it includes the centers of the Sun, the Earth and the spacecraft. The horizontal segment from the Earth's center to its surface points toward the center of the Sun. Arguments are measured counterclockwise, with the horizontal segment representing an argument of zero. β represents the argument of the spacecraft, while α represents the argument of the glint point (which must lie in the plane of the drawing.) The distance unit is one Earth radius, and λ represents the spacecraft's altitude.

II. Assumptions

The Sun is treated as though it were at infinite distance, and its rays of light are assumed to be parallel to one another. The Earth is treated as though it were perfectly circular, and the oceans as though they would reflect like mirrors. No atmospheric refractive effects are assumed.

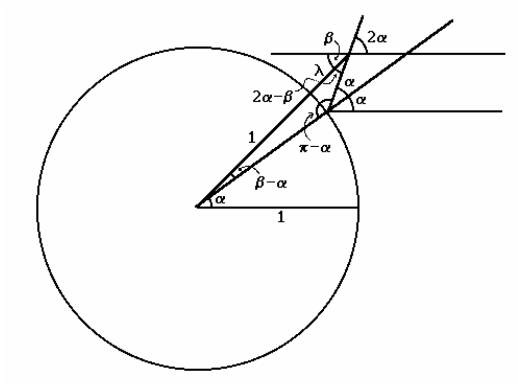

Figure 1. The Trigonometry of the Glint Problem.

III. Analysis

The Law of Sines implies

$$\frac{\sin(2\alpha-\beta)}{1}=\frac{\sin(\pi-\alpha)}{1+\lambda}$$

or more simply

$$\sin(2\alpha-\beta)=\frac{\sin\alpha}{1+\lambda}$$

$$\cos(2\alpha-\beta)=\sqrt{1-\frac{\sin^2\alpha}{(1+\lambda)^2}}$$

$$\frac{\sin\alpha}{1+\lambda}\cos\beta+\sqrt{1-\frac{\sin^2\alpha}{(1+\lambda)^2}}\sin\beta=\sin(2\alpha-\beta)\cos\beta+\cos(2\alpha-\beta)\sin\beta$$

$$\frac{\sin\alpha}{1+\lambda}\cos\beta+\sqrt{1-\frac{\sin^2\alpha}{(1+\lambda)^2}}\sin\beta=\sin 2\alpha$$

$$\frac{\sin\alpha}{1+\lambda}\cos\beta+\sqrt{1-\frac{\sin^2\alpha}{(1+\lambda)^2}}\sin\beta=2\sin\alpha\cos\alpha$$

$$\frac{\sin\alpha}{1+\lambda}\cos\beta+\sqrt{1-\frac{\sin^2\alpha}{(1+\lambda)^2}}\sin\beta=2\sin\alpha\sqrt{1-\sin^2\alpha}$$

Writing $x = \sin^2 \alpha$, we get

$$\frac{\sqrt{x}}{1+\lambda}\cos\beta + \sqrt{1-\frac{x}{(1+\lambda)^2}}\,\sin\beta = 2\sqrt{x}\sqrt{1-x}$$

Rearranging yields

$$\sqrt{1-\frac{x}{(1+\lambda)^2}}\,\sin\beta = 2\sqrt{x}\sqrt{1-x} - \frac{\sqrt{x}}{1+\lambda}\cos\beta$$

Squaring both sides, we get

$$\left(1-\frac{x}{(1+\lambda)^2}\right)\sin^2\beta = 4(x-x^2) - \frac{4\cos\beta}{1+\lambda}\sqrt{x^2-x^3} + \frac{\cos^2\beta}{(1+\lambda)^2}x$$

$$\frac{4\cos\beta}{1+\lambda}\sqrt{x^2-x^3} = 4\left(x-x^2\right) - \left(1-\frac{x}{(1+\lambda)^2}\right)\sin^2\beta + \frac{\cos^2\beta}{(1+\lambda)^2}x$$

$$= -4x^2 + \left(4+\frac{\cos^2\beta}{(1+\lambda)^2}+\frac{\sin^2\beta}{(1+\lambda)^2}\right)x - \sin^2\beta$$

$$= -4x^2 + \left(4+\frac{1}{(1+\lambda)^2}\right)x - \sin^2\beta$$

Again squaring both sides, we get

$$\frac{16\cos^2\beta}{(1+\lambda)^2}\left(x^2-x^3\right) = 16\left(x^2 - \left(1+\frac{1}{4(1+\lambda)^2}\right)x + \tfrac{1}{4}\sin^2\beta\right)^2$$

$$\frac{\cos^2\beta}{(1+\lambda)^2}\left(x^2-x^3\right) = x^4 + \left(1+\frac{1}{4(1+\lambda)^2}\right)^2 x^2 + \tfrac{1}{16}\sin^4\beta - 2\left(1+\frac{1}{4(1+\lambda)^2}\right)x^3 + \tfrac{1}{2}\left(\sin^2\beta\right)x^2$$
$$-\tfrac{1}{2}\left(1+\frac{1}{4(1+\lambda)^2}\right)\left(\sin^2\beta\right)x$$

The Quartic Equation

The foregoing analysis yields the following quartic equation in x:

$$x^4 + \left\{ \frac{\cos^2 \beta}{(1+\lambda)^2} - 2\left(1 + \frac{1}{4(1+\lambda)^2}\right) \right\} x^3 + \left\{ -\frac{\cos^2 \beta}{(1+\lambda)^2} + \left(1 + \frac{1}{4(1+\lambda)^2}\right)^2 + \tfrac{1}{2}\sin^2 \beta \right\} x^2$$

$$+ \left\{ -\tfrac{1}{2}\left(1 + \frac{1}{4(1+\lambda)^2}\right)(\sin^2 \beta) \right\} x + \left\{ \tfrac{1}{16}\sin^4 \beta \right\} = 0$$

For each real root x_i satisfying $0 \le x_i \le 1$, there is a unique corresponding candidate α_i such that $0 \le \alpha_i \le \frac{\pi}{2}$ and $x_i = \sin^2 \alpha_i$, i.e. $\alpha_i = \arcsin \sqrt{x_i}$. Upon testing each of these (up to four) candidate α_i 's in the equation $\sin(2\alpha - \beta) = \dfrac{\sin \alpha}{1+\lambda}$, one will be found to fit.